Open-Ended RC Line Model Predicts MOSFET IC Response

Propagation delay time in a string of MOS Gates may surprise you by being 100% better than you expected.

AMOS WILNAI, Signetics Corp.

A common problem many designers face in modern MOS integrated circuits is how fast a distributed-parameter open-ended RC line will respond to a step voltage excitation. Such a line is constructed when a conducting line is interconnecting many MOS gates, e.g., X or Y select lines in static RAMs, read or write select lines in dynamic RAMs, decode buses, etc. Usually a digital circuit designer is not interested so much in the frequency response equations of the line as in the time response of the line to pulse excitation. In the case of a metal line, with relatively low internal resistance, the time response of the line is usually fast enough. Not so if diffused bed undercrossings are employed in the line or where polysilicon interconnecting lines are being used (as in the case of modern polysilicon gate structures). Propagation delay through such lines can be significant.

We will attempt to determine the response of such a line to a step function excitation. This will give a good idea as to what time delay we should expect before such a line reaches a given voltage. Also having 2.2 *RC* in mind (the time it takes a lumped RC circuit to rise from 10% to 90% of its final voltage), we will find a corresponding number for the distributed line.

The Laplace transformation of the step response of a leakage free, non-inductive open-ended distributed line with lumped time constant *RC* (**Fig. 1**) is found to be

$$V_{out(s)} = \frac{1}{s \cosh \sqrt{sRC}} \qquad (1)$$

when *R* and *C* are the LUMPED values of the line resistance and capacitance, respectively.

The inverse transformation (the real time response) could not be found in close form. We will, therefore, try to arrive at an approximation of the exact response. Recalling that

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$= 1 + \frac{x^2}{2\,!} + \frac{x^4}{4\,!} + \frac{x^6}{6\,!} + \ldots \qquad (2)$$

$$\text{for } Re\, x \gg 1 \qquad \cosh x \simeq \frac{e^x}{2} \qquad (3)$$

for $Re\, x < 1$

$$\cosh x \simeq 1 + \frac{x^2}{2\,!} + \frac{x^4}{4\,!} \qquad (4)$$

for $Re \sqrt{sRC} \gg 1$ (the response to the high frequency section—leading edge—of the step function), the trans-

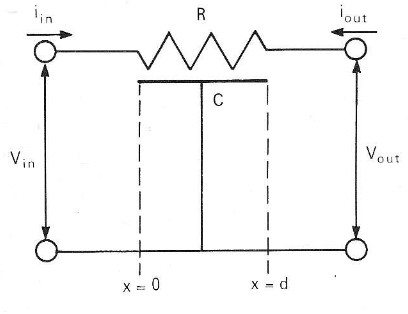


Fig. 1—**Lumped-value** form of an open-ended RC distributed line from which Laplace transforms were derived to approximate pulse-response times.

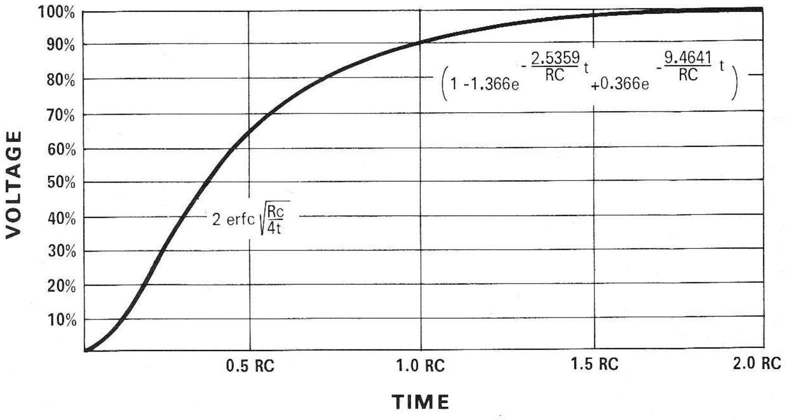
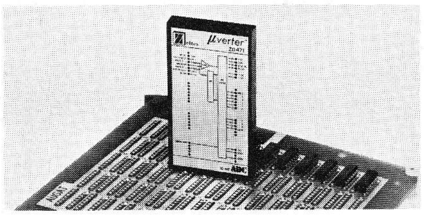


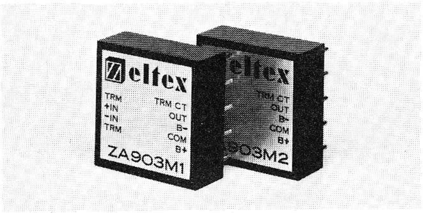
Fig. 2—**Response time** of the distributed RC line as calculated from the inverse Laplace.

(Continued)

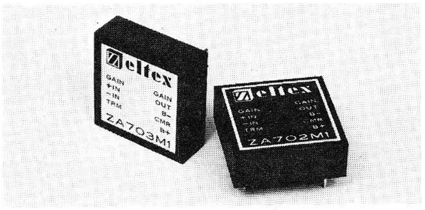
FOR A FREE REPRINT OF THIS ARTICLE, **CIRCLE NO. L66** new products



NEW 10-BIT ADC
Model ZD 471 offers 10-Bit resolution and 30 µs conversion time with 0.1% accuracy. Features include: TTL/DTL; ±10V, ±5V, and 0 to +10V operation over 0° to 70°C; adjustable FSR and zero offset; built-in reference stability – 100µV/30 days; and bake-in at 85°C for 16 hours for optimum reliability and stability. Data output is NRZ serial or parallel binary bits. Package is only 0.4 inches high and fits standard dip sockets. Availability: Stock
Price (1-9) – $99 Circle Service No. 41



1µV/°C FET AMPLIFIER
New model ZA903M2 features ultra-low voltage drift of 1µV/°C which is independent of zero offset adjustments. Other features include: 10pA input current, 4MHz frequency response and 100 KHz full power output. Package is low profile (1X1X0.4 high). Economy Model ZA903M1 features 3µV/°C. Availability: Stock
Price (OEM) – $39 Circle Service No. 42



LOW COST INSTRUMENTATION AMPLIFIER
Model ZA703M1 offers FET input stage and gain range of 1-1000 selected with only one external resistor. Input current is 3pA with gain non-linearity of 0.02%. Package is only 0.4 inches high. Model ZA702M1 (bipolar input) also available. Availability: Stock
Price (1-9) – $29 Circle Service No. 43

Call, TWX, or write today!
TEL: (415)686-6660
TWX: 910-481-9477

Zeltex INC.
A SUBSIDIARY OF REDCOR CORP.
1000 Chalomar Road Concord, California 94520

RC Line (Cont'd)

formed step response becomes

$$V_{out(s)} = 2\,\frac{e^{-\sqrt{sRC}}}{s} \qquad (5)$$

and the time response is found in standard Laplace transforms table to be

$$v_{out(t)} = 2\ erfc\ \sqrt{\frac{RC}{4t}} \qquad (6)$$

for $Re\ \sqrt{sRC} \ll 1$ (the response to the low frequency section of the step function) the transformed step response becomes

$$V_{out(s)} = \frac{1}{s\left(1 + \frac{sRC}{2\,!} + \frac{(sRC)^2}{4\,!}\right)} = \frac{24/\ (RC)^2}{s\left(s + \frac{2.5359}{RC}\right)\left(s - \frac{9.4641}{RC}\right)} \qquad (7)$$

with time response

$$v_{out(t)} = 1 - 1.3660\,e^{-\frac{2.5359}{RC}t} + 0.3660\,e^{-\frac{9.4641}{RC}t} \qquad (8)$$

The complete approximate response based on **Eqs. 6** and **8** is plotted in **Fig. 2**.

Note that the time it takes the line to reach from 0 to 90% of its final voltage is 1.0 *RC*; from 10% to 90% (rise time) is 0.89 *RC*; and from 0 to 10% (delay time) is 0.13 *RC*. Compare that with the 2.3 *RC* it takes a corresponding lumped RC circuit to rise from 0 to 90%; 2.2 *RC* from 10% to 90%, and 0.11 *RC* from 0 to the 10% point.

Experimental Results. Because of the relative difficulty of constructing and measuring a purely distributed RC line, we tried to simulate such a line with an increasing number of discrete RC sections. The plot of the time it took these circuits to rise from 0 and 10% to the 90% point of their final value is shown in **Fig. 3**. Here the transition time in total lumped RC units is plotted as a function of the number of discrete RC sections from which the line was constructed. Note the conversion of the graph to the 1 RC line. ☐

Amos Wilnai was employed at Signetics Corp. when he prepared this article. He has since moved to Monolithic Memories, Inc. in Sunnyvale, Calif. where he is in IC design.

He holds a B.S.E.E. from Technion, Israel Institute of Technology in Haifa, Israel and a M.S.E.E. from the Polytechnic Institute of Brooklyn. He has done additional graduate work at the University of California at Berkeley.

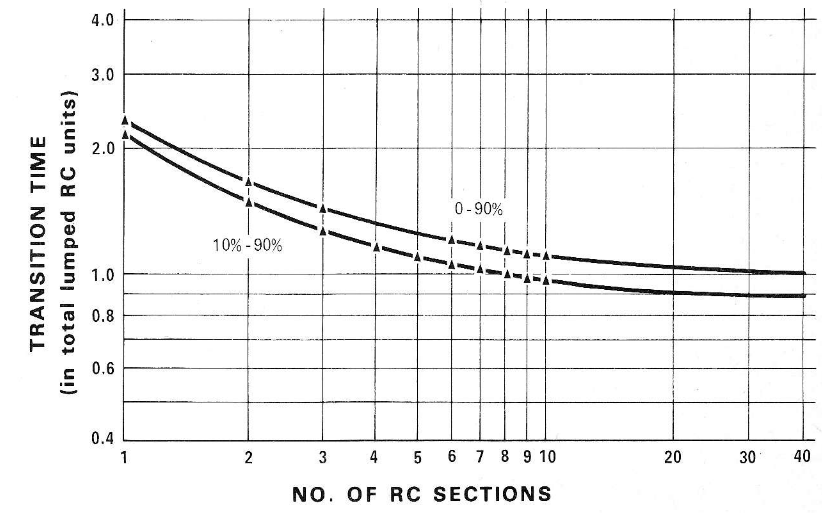


Fig. 3 – **Empirical results** clearly depict that response time is not the normally anticipated 2.2 RC, but approaches 1.0 RC as predicted by the author.

FOR A FREE REPRINT OF THIS ARTICLE, **CIRCLE NO. L66**